

An understanding for flavor physics in the lepton sector

Zhen-hua Zhao

Institute of Theoretical Physics, Chinese Academy of Sciences,

and State Key Laboratory of Theoretical Physics,

*P. O. Box 2735, Beijing 100190, China**

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Abstract

In this paper, we give a model for understanding flavor physics in the lepton sector, mass hierarchy among different generations and neutrino mixing pattern. The model is constructed in the framework of supersymmetry, with a family symmetry $S4 \times Z19$. There is only one right-handed neutrino introduced for seesaw mechanism, while some SM singlet fields are included which transforms non-trivially under family symmetry. In the model, each order of contributions are suppressed by $\delta \sim 0.1$ compared to the previous one. In order to reproduce the mass hierarchy, m_τ and $\sqrt{\Delta m_{atm}^2}$, m_μ and $\sqrt{\Delta m_{sol}^2}$ are obtained at leading-order(LO) and next-to-leading-order(NLO) respectively, while electron can only get its mass through NNNLO contributions. For neutrino mixing angles, $\theta_{12}, \theta_{23}, \theta_{13}$ are $45^\circ, 45^\circ, 0$ i.e. Bi-maximal mixing pattern as first approximation, while higher order contributions can make them consistent with experimental results. As corrections for θ_{12} and θ_{13} originate from the same contribution, there is a relation predicted for them $\sin \theta_{13} = \frac{1 - \tan \theta_{12}}{1 + \tan \theta_{12}}$. Besides, deviation from $\frac{\pi}{4}$ for θ_{23} is very small which is order 0.01.

*Electronic address: zhzhao@itp.ac.cn

I. INTRODUCTION

Before, many believed that θ_{13} is very small due to the fact that experiments could only give an upper bound for it for a long time. In this situation, the scenario with the so-called Tri-bimaximal mixing pattern [6] which gives a vanishing θ_{13} as LO approximation was very popular. Remarkably, it is found that many models with a discrete flavor symmetry can realize this mixing pattern from an underlying theory [7, 8]. However, considering latest experimental results for θ_{13} [1–5] which turned out to be much larger than many expected, Tri-bimaximal model needs substantial modification. The main problem with it lies in that it is difficult to understand how and why NLO corrections take θ_{13} from 0 to a rather large angle while preserving θ_{12} and θ_{23} close to their values given at LO.

In this paper, we would like to give a simple alternative to Tri-bimaximal model. For convenience of expressing our ansatz for mass matrices of leptons explicitly, a model realizing our ansatz is given. In the following, this realistic model is given directly, while observations about lepton flavor physics are given implicitly under discussions for the model. In the model, mass hierarchy in the lepton sector as well as neutrino mixing pattern are natural results. To reproduce the mass hierarchy, fermion masses are produced in different orders of contributions. Because of special forms for mass matrices, realistic neutrino mixing pattern will also be obtained. However, it should be emphasized that we really would like to present is not the model itself but the understanding for lepton flavor physics.

II. THE MODEL

The model is built in the framework of supersymmetry with family symmetry $S4 * Z19$; field contents and their transformation properties under $S4 * Z19 * U(1)_R$ are given in Table 1. $S4$ group has 2 singlet representations, 1 doublet representations and 2 triplet representations which are denoted as 1, 1', 2, 3 and 3' in order. For its presentation, we will adopt the one reported in appendix of [9] where readers can find details about multiplication rules and C-G coefficients. (For a recent review about models for flavor physics with $S4$ family symmetry, please see [10].) As shown in Table 1, it is arranged that three $SU(2)_L$

	L	e^c	(μ^c, τ^c)	N	$H_{u,d}$	Ψ	Ω	Σ	Φ	Π^0	Θ^0
$S4$	3	1	2	1	1	$3'$	3	2	$3'$	2	3
$Z19$	$\frac{17}{19}$	$\frac{3}{19}$	$\frac{16}{19}$	1	1	$\frac{1}{19}$	$\frac{2}{19}$	$\frac{3}{19}$	$\frac{5}{19}$	$\frac{11}{19}$	$\frac{16}{19}$
$U(1)_R$	0	1	1	1	1	0	0	0	0	2	2

TABLE I: Transformation properties of all the fields under $S4 * Z19 * U(1)_R$.

doublets combine to be representation 3 under $S4$, e^c and (μ^c, τ^c) transform in the way 1 and 2 respectively, while right handed neutrino N is in trivial representation 1. Higgs fields $H_{u,d}$ are trivial representations under family symmetry, while some SM singlet flavon fields which fall in non-trivial representations of $S4$ are included.

Due to the transformation properties distributed above, LO terms that can generate masses for leptons are of dimension 5. Besides, only when flavon fields get vacuum expectation values (VEVs) can fermion masses be produced. In this situation, terms contributing to fermion masses are characterized by $(\frac{v}{M})^n$ where v denotes flavon fields' VEVs, M is cutoff scale for flavon physics and n indicates number of flavon fields. In following discussions, we assume that $\delta = \frac{v}{M} \sim 0.1$. As a result, the order a term belongs to can be classified by the number of flavon fields.

A cyclic $Z19$ symmetry is also introduced under which fields' transformation property are denoted by multiples of $\frac{1}{19}$. With family symmetry $Z19$, appropriate flavon fields can be picked out for different leptons to produce mass matrices that is needed. Besides, R-symmetry is included which plays a key part in discussing flavon fields' VEVs. As flavon fields have 0 charge under R-symmetry, they have to appear in companion with a driving field which is marked with a suffix 0 in Table 1. Consequently, supersymmetric condition that F components of driving fields cannot have VEVs provides constraints on flavon fields' VEVs.

For the time being, we just assume flavon fields' VEVs have the following form and are stable against higher order contributions,

$$\langle \Psi \rangle = \begin{pmatrix} v_1 \\ 2v_1 \\ 0 \end{pmatrix} \quad \langle \Omega \rangle = \begin{pmatrix} 0 \\ v_2 \\ 0 \end{pmatrix} \quad \langle \Sigma \rangle = \begin{pmatrix} v_3 \\ 0 \end{pmatrix} \quad \langle \Phi \rangle = \begin{pmatrix} 0 \\ v_4 \\ v_4 \end{pmatrix}, \quad (1)$$

where all the VEVs are assumed to be close to each other $v_1 \sim v_2 \sim v_3 \sim v_4 \sim v$.

A. Physics at Leading Order

At LO, superpotential includes the following terms that contribute to lepton masses,

$$\frac{1}{\Lambda} y_1 [(\mu^c, \tau^c) L]_{3'} \Phi H_d + \frac{1}{\Lambda} y_2 N L \Omega H_u + M N N, \quad (2)$$

where we use $[\quad]_{3'}$ to indicate that $(\mu^c, \tau^c) L$ combines to become representation $3'$ and so on. In this work, all dimensionless coupling such as y_1 and y_2 in Eq.(2) are assumed to be order 1 and close to each other. With VEVs in Eq.(1), mass matrices for charged leptons and light neutrinos are as follows,

$$M_e = \frac{y_1 v_d v_4}{\Lambda} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}, \quad M_\nu = \frac{(y_1 v_u v_2)^2}{M \Lambda^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (3)$$

$M_e^\dagger M_e$ have the following form,

$$M_e^\dagger M_e = \left(\frac{y_1 v_d v_4}{\Lambda} \right)^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}. \quad (4)$$

Thus, only tau and one neutrino have non-zero masses $m_\tau = \sqrt{2} \frac{y_1 v_d v_4}{M}$ and $m_3 = \frac{(y_1 v_u v_2)^2}{M \Lambda^2}$.

The matrix that diagonalize Eq.(4) has a simple form,

$$U_e = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (5)$$

while U_ν is simply identity matrix at this order.

B. Physics at Next to Leading Order

After taking NLO contributions into consideration, there are new terms contributing to lepton masses,

$$\frac{1}{\Lambda^2} y_3 [(\mu^c, \tau^c) L]_3 [\Omega \Sigma]_3 H_d + \frac{1}{\Lambda^2} y_4 [(\mu^c, \tau^c) L]_{3'} [\Omega \Sigma]_{3'} H_d + \frac{1}{\Lambda^2} y_5 N L [\Psi \Psi]_3 H_u. \quad (6)$$

As a result, charged lepton and light neutrino mass matrices become,

$$M'_e = \frac{y_1 v_d v_4}{\Lambda} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} + \frac{\delta_1}{4} + \frac{3\delta_2}{4} \\ 0 & \frac{1}{2} - \frac{\sqrt{3}\delta_1}{4} + \frac{\sqrt{3}\delta_2}{4} & \frac{1}{2} \end{pmatrix}, \quad (7)$$

where $\delta_1 = \frac{y_3 v_2 v_3}{y_1 v_4 \Lambda} \sim \delta_2 = \frac{y_4 v_2 v_3}{y_1 v_4 \Lambda} \sim \delta$;

$$M'_\nu = \frac{(y_1 v_u v_2)^2}{M \Lambda^2} \begin{pmatrix} 16\delta_3^2 & 16\delta_3^2 & 0 \\ 16\delta_3^2 & 16\delta_3^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (8)$$

where $\delta_3 = \frac{y_5 v_1 v_1}{y_2 v_2 \Lambda} \sim \delta$.

There will be no higher orders of contributions to neutrino mass, so Eq.(8) is the final result. The following matrix diagonalize Eq.(9) to obtain three mass eigenvalues $m_1 = 0, m_2 = 32\delta_3^2 \frac{(y_1 v_u v_2)^2}{M \Lambda^2}, m_3 = \frac{(y_1 v_u v_2)^2}{M \Lambda^2}$,

$$U'_\nu = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (9)$$

With experimental results for neutrino oscillations [12], $\frac{m_2}{m_3} = 32\delta_3^2 = \sqrt{\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}} = 0.18$. Noteworthy, in mass matrix for light neutrinos, NLO contributions should have been 2 orders smaller than LO ones after seesaw mechanism. The factor 16 arising from C-G coefficients in Eq.(8) plays a crucial role in making $\frac{m_2}{m_3}$ consistent with experimental result.

Intuitively, smaller eigenvalue of $M_e'^\dagger M_e'$ should be about $\delta(\frac{y_1 v_d v_4}{\Lambda})^2$. However, it can be proved that $\det(M_e'^\dagger M_e') = 0$ up to NLO, due to special form of M_e . Further, $\det(M_e'^\dagger M_e') \neq 0$ up to NNLO, so smaller eigenvalue of $M_e'^\dagger M_e'$ is about $\delta^2(\frac{y_1 v_d v_4}{\Lambda})^2$ while the larger one remains about $2(\frac{y_1 v_d v_4}{\Lambda})^2$. Thus, $\frac{m_\mu}{m_\tau} \approx \sqrt{\frac{\delta^2}{2}}$, compatible with experimental result 0.06. This time, the fact $\det(M_e'^\dagger M_e') = 0$ at NLO help us avoid parameters' fine-tuning to be compatible with experimental results. As for U_e' , only θ_{23} is non-zero and there is an estimate for it,

$$\tan 2\theta_{23} \approx \frac{2}{\delta}. \quad (10)$$

If we parameterize deviation from $\frac{\pi}{4}$ for θ_{23} with a small quantity ϵ_1 , it is about $\frac{\delta}{4}$ and U_e' can be described in the following form,

$$U_e' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}}(1 + \epsilon_1) & \frac{1}{\sqrt{2}}(1 - \epsilon_1) \\ 0 & -\frac{1}{\sqrt{2}}(1 - \epsilon_1) & \frac{1}{\sqrt{2}}(1 + \epsilon_1) \end{pmatrix}. \quad (11)$$

C. Physics at Next-to-Next-to-Leading-Order

New terms that contribute to lepton masses are listed below,

$$\begin{aligned} & \frac{1}{\Lambda^3} y_6 [(\mu^c, \tau^c) L]_3 [(\Omega\Omega)_2 \Psi]_3 H_d + \frac{1}{\Lambda^3} y_7 [(\mu^c, \tau^c) L]_3 [(\Omega\Omega)_3 \Psi]_3 H_d \\ & + \frac{1}{\Lambda^3} y_8 [(\mu^c, \tau^c) L]_{3'} [(\Omega\Omega)_2 \Psi]_{3'} H_d + \frac{1}{\Lambda^3} y_9 [(\mu^c, \tau^c) L]_3 [\Sigma(\Psi\Psi)_3]_3 H_d. \end{aligned} \quad (12)$$

At this stage, mass matrix for charged leptons becomes,

$$M_e' = \frac{y_1 v_d v_4}{\Lambda} \begin{pmatrix} 0 & 0 & 0 \\ -\frac{\sqrt{3}}{2} \delta_4^2 - 4\delta_5^2 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} + \frac{\delta_1}{4} + \frac{3\delta_2}{4} \\ 0 & \frac{1}{2} - \frac{\sqrt{3}\delta_1}{4} + \frac{\sqrt{3}\delta_2}{4} & \frac{1}{2} \end{pmatrix}, \quad (13)$$

where $\delta_4^2 = \frac{y_6 v_1 v_2 v_2}{y_1 v_4 \Lambda \Lambda} \sim \delta_5^2 = \frac{y_9 v_1 v_1 v_3}{y_1 v_4 \Lambda \Lambda} \sim \delta^2$. In Eq.(13), we have neglected contributions to those matrix elements which are non-zero at NLO.

In order to make physics clear, we do a qualitative analysis for U_e'' up to matrix elements'

orders. First of all, we effect transformation U'_e on $M_e''^\dagger M_e''$,

$$U_e'^\dagger M_e''^\dagger M_e'' U_e' \approx \frac{y_1 v_d v_4}{\Lambda} \begin{pmatrix} \delta^4 & \delta^3 & \delta^2 \\ \delta^3 & \delta^2 & 0 \\ \delta^2 & 0 & 2 \end{pmatrix}. \quad (14)$$

To diagonalize Eq.(14), we just need to effect a transformation with $\sin \theta_{13} \sim \delta^2$ and a transformation with $\sin \theta_{12} \sim \delta$ successively. If we ignore the negligibly small θ_{13} and parameterize θ_{12} with another small quantity ϵ_2 , U_e'' have the following form,

$$U_e'' = \begin{pmatrix} 1 & \epsilon_2 & 0 \\ -\frac{\epsilon_2}{\sqrt{2}} & \frac{1}{\sqrt{2}}(1 + \epsilon_1) & \frac{1}{\sqrt{2}}(1 - \epsilon_1) \\ \frac{\epsilon_2}{\sqrt{2}} & -\frac{1}{\sqrt{2}}(1 - \epsilon_1) & \frac{1}{\sqrt{2}}(1 + \epsilon_1) \end{pmatrix}. \quad (15)$$

At present, we can discuss about neutrino mixing angles. U_{PMNS} [11] is obtained by $U_e''^\dagger U'_\nu$,

$$U = \begin{pmatrix} \frac{1}{\sqrt{2}}(1 + \frac{1}{\sqrt{2}}\epsilon_2) & \frac{1}{\sqrt{2}}(1 - \frac{1}{\sqrt{2}}\epsilon_2) & \frac{\epsilon_2}{\sqrt{2}} \\ \dots & \dots & \frac{1}{\sqrt{2}}(1 - \epsilon_1) \\ \dots & \dots & \frac{1}{\sqrt{2}}(1 + \epsilon_1) \end{pmatrix}. \quad (16)$$

where we just list the matrix elements involved in fixing neutrino mixing angles. In this

case, $\tan \theta_{12} = \frac{1 - \frac{1}{\sqrt{2}}\epsilon_2}{1 + \frac{1}{\sqrt{2}}\epsilon_2}$, $\sin \theta_{13} = \frac{\epsilon_2}{\sqrt{2}}$, $\tan \theta_{23} = \frac{1 - \epsilon_1}{1 + \epsilon_1}$. The larger the deviation from $\frac{\pi}{4}$

for θ_{12} the larger θ_{13} is, resulting from the relation $\sin \theta_{13} = \frac{1 - \tan \theta_{12}}{1 + \tan \theta_{12}}$. Unfortunately, the above relation is just consistent with experimental results within 2σ [1, 12]. Considering ϵ_1 is about $\frac{\delta}{4}$, θ_{23} will remain close to 45° .

D. Physics at Next-to-Next-to-Next-to-Leading-Order

Electron cannot obtain its mass until this order where interacting terms between e^c and L appears for the first time,

$$\begin{aligned} & \frac{1}{\Lambda^4} [(\mu^c, \tau^c) L]_3 H_d \{ y_{10} [(\Phi\Phi)_1 (\Sigma\Phi)_3]_3 + y_{11} [(\Phi\Phi)_2 (\Sigma\Phi)_3]_3 + y_{12} [(\Phi\Phi)_2 (\Sigma\Phi)_{3'}]_3 \\ & + y_{13} [(\Phi\Phi)_3 (\Sigma\Phi)_3]_3 + y_{14} [(\Phi\Phi)_3 (\Sigma\Phi)_{3'}]_{3'} + y_{15} [(\Phi\Phi)_{3'} (\Sigma\Phi)_3]_3 + y_{16} [(\Phi\Phi)_{3'} (\Sigma\Phi)_{3'}]_{3'} \}. \end{aligned} \quad (17)$$

Eq.(17) leads to a naturally small electron mass, without interfering the above discussions.

E. Flavon Fields' VEVs

Finally, we would like to address issues concerning VEV alignments in Eq.(1) which needs to be a reasonable result in order to make this model convincing. We just need to show that these VEV alignments are valid up to NNLO, because our physical results except electron mass have been achieved by this order while production of electron mass does not rely on those specific VEV alignments. As we have said, supersymmetric requirements result in constraint on F components of driving fields Π^0 and Θ^0 that $\langle F_i \rangle = 0$ with i representing $\Pi_1^0, \Pi_2^0, \Theta_1^0, \Theta_2^0, \Theta_3^0$. Relevant terms are given below:

$$\begin{aligned}
\text{LO,} \quad & \Pi^0 \{m\Sigma + c_1(\Omega\Psi)_2\} + \Theta^0 \{c_4(\Phi\Phi)_3\}; \\
\text{NLO,} \quad & \frac{1}{\Lambda} \Pi^0 \{c_2[(\Psi\Psi)_3\Psi]_2 + c_3[(\Psi\Psi)_{3'}\Psi]_2\} + \frac{1}{\Lambda} \Theta^0 \{c_5[(\Sigma\Phi)_3\Omega]_3 + c_6[(\Sigma\Phi)_{3'}\Omega]_3\}; \\
\text{NNLO,} \quad & \frac{1}{\Lambda^2} \Theta^0 \{c_7\{[(\Sigma\Phi)_3\Psi]_{1'}\Psi\}_3 + c_8\{[(\Sigma\Phi)_3\Psi]_2\Psi\}_3 + c_9\{[(\Sigma\Phi)_3\Psi]_3\Psi\}_3 \\
& + c_{10}\{[(\Sigma\Phi)_3\Psi]_{3'}\Psi\}_3 + c_{11}\{[(\Sigma\Phi)_{3'}\Psi]_2\Psi\}_3 + c_{12}\{[(\Sigma\Phi)_{3'}\Psi]_3\Psi\}_3 + c_{13}\{[(\Sigma\Phi)_{3'}\Psi]_{3'}\Psi\}_3 \\
& + c_{14}\{[(\Omega\Omega)_1\Phi]_{3'}\Psi\}_3 + c_{15}\{[(\Omega\Omega)_2\Phi]_3\Psi\}_3 + c_{16}\{[(\Omega\Omega)_2\Phi]_{3'}\Psi\}_3 + c_{17}\{[(\Omega\Omega)_3\Phi]_{1'}\Psi\}_3 \\
& + c_{18}\{[(\Omega\Omega)_3\Phi]_2\Psi\}_3 + c_{19}\{[(\Omega\Omega)_3\Phi]_3\Psi\}_3 + c_{20}\{[(\Omega\Omega)_3\Phi]_{3'}\Psi\}_3 + c_{21}\{[(\Omega\Omega)_{3'}\Phi]_2\Psi\}_3 \\
& + c_{22}\{[(\Omega\Omega)_{3'}\Phi]_3\Psi\}_3 + c_{23}\{[(\Omega\Omega)_{3'}\Phi]_{3'}\Psi\}_3 + c_{24}\{[(\Sigma\Sigma)_1\Sigma]_2\Psi\}_3 + c_{25}\{[(\Sigma\Sigma)_{1'}\Sigma]_2\Psi\}_3 \\
& + c_{26}\{[(\Sigma\Sigma)_2\Sigma]_{1'}\Psi\}_3 + c_{27}\{[(\Sigma\Sigma)_2\Sigma]_2\Psi\}_3 + c_{28}\{[(\Sigma\Sigma)_1\Omega]_3\Omega\}_3 + c_{29}\{[(\Sigma\Sigma)_{1'}\Sigma]_{3'}\Psi\}_3 \\
& + c_{30}\{[(\Sigma\Sigma)_2\Sigma]_3\Psi\}_3 + c_{31}\{[(\Sigma\Sigma)_2\Sigma]_{3'}\Psi\}_3\}.
\end{aligned}$$

There are totally 13 equations for $\langle F_i \rangle = 0$ at LO, NLO, NNLO respectively, among which 8 equations are automatically satisfied when flavon fields take VEVs as shown in Eq.(1). The other 5 equations are as follows,

$$\left\{ \begin{array}{ll} mv_3 + \sqrt{3}c_1v_1v_2 & = 0 \\ (c_5 + \sqrt{3}c_6)v_2v_3v_4 & = 0 \\ \lambda_1v_1v_1v_3v_4 + \lambda_2v_1v_2v_2v_4 + \lambda_3v_2v_2v_3v_3 & = 0, \\ \lambda_4v_1v_1v_3v_4 + \lambda_5v_1v_2v_2v_4 & = 0 \\ \lambda_6v_1v_1v_3v_4 + \lambda_7v_1v_2v_2v_4 + \lambda_8v_1v_3v_3v_3 & = 0 \end{array} \right. \quad (18)$$

where $\lambda_1 = \sqrt{3}c_7 - \frac{\sqrt{3}}{2}c_8 - \sqrt{3}c_9 - \sqrt{3}c_{10} + \frac{\sqrt{3}}{2}c_{11} + c_{12} + c_{13}$,

$$\begin{aligned}
\lambda_2 &= \frac{\sqrt{3}}{2}c_{15} - \frac{3}{2}c_{16} - 2c_{19} + 2c_{20}, \\
\lambda_3 &= -c_{28} - \frac{1}{2}c_{30} + \frac{\sqrt{3}}{2}c_{31}, \\
\lambda_4 &= 2\sqrt{3}c_7 + \frac{\sqrt{3}}{2}c_8 - \frac{3\sqrt{3}}{2}c_9 - \frac{\sqrt{3}}{2}c_{10} - \frac{\sqrt{3}}{2}c_{11} - \frac{5}{2}c_{12} + \frac{1}{2}c_{13}, \\
\lambda_5 &= \frac{\sqrt{3}}{4}c_{15} + \frac{3}{4}c_{16} + c_{19} + c_{20}, \\
\lambda_6 &= \frac{3\sqrt{3}}{2}c_8 + \frac{\sqrt{3}}{2}c_9 + \frac{3\sqrt{3}}{2}c_{10} + \frac{\sqrt{3}}{2}c_{11} - \frac{1}{2}c_{12} + \frac{5}{2}c_{13}, \\
\lambda_7 &= -\frac{\sqrt{3}}{4}c_{15} - \frac{3}{4}c_{16} + c_{19} + c_{20}, \\
\lambda_8 &= \sqrt{3}c_{24} + \sqrt{3}c_{27}.
\end{aligned}$$

For the second equation in Eq.(19), we have to assume an accidental relation $c_5 + \sqrt{3}c_6 = 0$. In this situation, values of $v_1 - v_4$ are determined from the other 4 equations and should be in the order of m which is the only coefficient that has dimension. Thus, we can say the assumption $v_1 \sim v_2 \sim v_3 \sim v_4 \sim v$ is reasonable.

III. CONCLUSIONS AND DISCUSSIONS

In conclusion, we have built a model for understanding flavor physics in the lepton sector, mass spectrum and mixing pattern. The model is constructed under family symmetry $S4 * Z19$. With the assumption that higher order contribution is suppressed by $\delta \sim 0.1$ compared to previous one, the mass hierarchy $\frac{m_e}{m_\mu}, \frac{m_\mu}{m_\tau}, \sqrt{\frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}}$ are natural results of this model. This is realized by producing m_τ, m_3 at LO, m_μ, m_2 at NLO and m_e at NNNLO while $m_1 = 0$ to all orders. In fact, this realization is to some extent inspired by Chun Liu's works [13] where electron, muon and tau get their mass from breaking of different symmetries. At the same process of reproducing mass spectrum, realistic mixing pattern is obtained (in Refs.[14], the authors also attempted to connect mixing angles with mass hierarchy). As a matter of fact, this model's mixing pattern for first approximation is actually Bi-maximal [15, 16]. There are some works [17–19] obtaining Bi-maximal mixing pattern with the same starting point as this model. As far as mixing angles are concerned, this work does not only provide a realistic model where higher order contributions change θ_{13} and θ_{12} considerably without interfering θ_{23} much, but also predicts a relation for θ_{13} and θ_{12} $\sin \theta_{13} = \frac{1 - \tan \theta_{12}}{1 + \tan \theta_{12}}$.

However, there are still two problems to solve in this model. The essential one is that the relation $\sin\theta_{13} = \frac{1 - \tan\theta_{12}}{1 + \tan\theta_{12}}$ is not very consistent with experimental results. The other one is concerned with the unnatural relation $c_5 + \sqrt{3}c_6 = 0$. In a word, this model provides an insight into lepton flavor physics and deserves further studies due to both its advantages and disadvantages.

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